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HEAT TRANSFER IN ENCLOSURES WITH PROGRAMMED HEAT RELEASE

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The problems of calculating the indoor air temperature, heat consumption, and room temperatures with arbitrary thermal perturbations are examined.

In estimating the efficiency of programmed heat release and optimizing it, it is necessary to calculate the thermal conditions indoors taking into account nonstationary heat-transfer processes. Including a large number of nonstationary thermal perturbations permits improving the temperature–humidity conditions indoors and decreasing the amount of heat used [1].

The difficulty of introducing thermal conditions operationally with the help of automated control systems increases considerably in the setup being examined. This is related to the fact that the known solutions [2, 3], obtained for harmonic, jumplike changes in $q(\tau)$, $t_0(\tau)$ and other actions, are either not accurate enough or greatly complicate the operational estimate of parameters, since in order to use the solutions, it is necessary to expand the heat use function $q(\tau)$, external temperature $t_0(\tau)$, and other functions in a Fourier series in each subsequent calculation. In this connection, it is useful to examine the solution of the problem of heat transfer indoors using splines [4, 5], which give a simpler algorithm for operational control of the temperature conditions.

The system of equations that describes heat transfer indoors can be represented in the following form: for the heat balance indoors in accordance with [1]

$$VC_p \frac{dt_i(\tau)}{d\tau} = -F_o k_o (t_i(\tau) - t_o(\tau)) - C_i(\tau)(t_i(\tau) - t_o(\tau)) + q(\tau) - \sum_j F_{oj}\alpha_{oij}(t_i(\tau) - t_{ocj}(\tau)) - \sum_j F_{ii}\alpha_{ij}(t_i(\tau) - t_{icj}(\tau)),$$

for heat transfer in the external walls [2, 6]

$$\frac{\partial T_{oj}(x, \tau)}{\partial \tau} = a_{oj}\frac{\partial^2 T_{oj}(x, \tau)}{\partial x^2}, \quad 0 < x < l_{oj}, \tag{2}$$

$$-\lambda_{oj}\frac{\partial T_{oj}(0, \tau)}{\partial x} = \alpha_o^j (t_i(\tau) - T_{oj}(0, \tau)), \tag{3}$$

$$\lambda_{oj}\frac{\partial T_{oj}(l_{oj}, \tau)}{\partial x} = \alpha_{oj}(t_o(\tau) - T_{oj}(l_{oj}, \tau)), \tag{4}$$

$$T_{oj}(x, 0) = T_{oj}(x). \tag{5}$$

An analogous system of equations is valid for the interior walls:

$$\frac{\partial T_{ij}(x, \tau)}{\partial \tau} = a_{ij}\frac{\partial^2 T_{ij}(x, \tau)}{\partial x^2}, \quad 0 < x < \frac{l_{ij}}{2}, \tag{6}$$

$$\lambda_{ij}\frac{\partial T_{ij}\left(\frac{l_{ij}}{2}, \tau\right)}{\partial x} = \alpha_{ij}\left(t_i(\tau) - T_{ij}\left(\frac{l_{ij}}{2}, \tau\right)\right), \tag{7}$$

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$$\frac{\partial T_{ij}(0,\ \tau)}{\partial x} = 0, \tag{8}$$

$$T_{ij}(x,\ 0) = T_{ij}(x). \tag{9}$$

In formulating the problem, we use the assumptions commonly adopted in engineering calculations: the air temperature at each moment in time is constant over the entire volume of the enclosure [1, 7]; the surface of any wall at any time is isothermal [7]; the humidity conditions in the enclosures do not change with time [7]; the inner and outer walls are reduced to equivalent single-layer walls [8]; the coefficient of heat transfer in the temperature range examined is assumed to be constant [1, 9].

The simultaneous analytic solution of Eqs. (1)-(9) in order to determine $t_o(\tau)$ as a function of $q(\tau)$, $t_o(\tau)$, $C_{in}(\tau)$ is a very complex problem, which is usually solved under certain assumptions. Thus, in [10], this problem was solved with $C_{in}(\tau)$ = const and $q(\tau)$, $t_i(\tau)$ varying harmonically. The operational methods used in [2, 9] permit obtaining a solution with $C_{in}(\tau)$ = const and arbitrary variation of $q(\tau)$, $t_i(\tau)$.

As a rule, the problem is considered for a single inner and outer walls; the addition of one more or several barriers greatly complicates the problem. Many researchers use grid method [7] for solving this type of system of equations, but these require large expenditures of machine time.

In our work, the system of equations being examined (1)-(9) was solved using splines [4, 5]. The use of splines for solving thermophysical problems was first brought to our attention by V. N. Bogoslovskii. Here the heat-conduction equations are solved only once independent of $t_i(\tau)$ and $t_o(\tau)$. Then, the use of recurrence relations permits finding $t_o(\tau)$ for known $t_i(\tau)$, $q(\tau)$, $C_{in}(\tau)$; in addition, new data on these quantities do not complicate the calculations.

In exactly the same way $q(\tau)$ is determined for a given function $t_o(\tau)$.

It is known [6] that the solution of Eqs. (6)-(9) for definite functions $t_i(\tau)$, $t_o(\tau)$ has the form

$$\begin{aligned} T_{ij}(x,\ \tau) = \sum_{k=1}^{\infty} \Bigg\{ & \left(\cos z_{ikj}x + \alpha_{ioj}\frac{\sin z_{ikj}x}{\lambda_{ij}z_{ikj}}\right)\left\{\int_0^{l_{ij}} T_{ij}(\zeta)\left(\cos z_{ikj}\zeta + \alpha_{ioj}\frac{\sin z_{ikj}\zeta}{\lambda_{ij}z_{ikj}}\right)d\zeta \times \right. \\ & \times \exp(-a_{ij}z_{ikj}^2\tau) + \frac{\alpha_{ioj}a_{ij}(z_{ikj}^2\lambda_{ij}^2 + \alpha_{ioj}^2)}{\lambda_{ij}(z_{ikj}^2\lambda_{ij}^2 - \alpha_{ioj}\alpha_{iwj})}\cos z_{ikj}l_{ij}\int_0^{\tau} t_i(\theta)\exp(-a_{ij}z_{ikj}^2(\tau-\theta))\,d\theta + \\ & \left. + \frac{\alpha_{ioj}a_{ij}}{\lambda_{ij}}\int_0^{\tau} t_i(\theta)\exp(-a_{ij}z_{ikj}^2(\tau-\theta))\,d\theta\right\}\Bigg\}\left\{\frac{\alpha_{iwj}(\lambda_{ij}^2z_{ikj}^2 + \alpha_{ioj}^2)}{2\lambda_{ij}(\lambda_{ij}^2z_{ikj}^2 + \alpha_{iwj}^2)z_{ikj}^2} + \frac{\alpha_{ioj}}{2\lambda_{ij}z_{ikj}^2} + \frac{l_{ij}}{2}\left(1 + \frac{\alpha_{ioj}^2}{\lambda_{ij}^2z_{ikj}^2}\right)\right\}^{-1}, \end{aligned} \tag{10}$$

where z_{ikj} are the positive roots of the equation

$$\frac{\operatorname{tg} zl_{ij}}{z} = \frac{\lambda_{ij}(\alpha_{ioj} + \alpha_{iwj})}{z^2\lambda_{ij}^2 - \alpha_{ioj}\alpha_{iwj}}; \tag{11}$$

$$\begin{aligned} T_{oj}(x,\ \tau) = \sum_{k=1}^{\infty}(\cos z_{okj}x)\left[\frac{\alpha_{oj}}{2\lambda_{oj}z_{okj}^2}\left(1 + \frac{\alpha_{oj}}{2\lambda_{oj}}\right) + \frac{l_{oj}}{4}\right]^{-1}\Bigg[\int_0^{1/2\,l_{oj}} T_{oj}(\zeta)\cos z_{okj}\zeta\,d\zeta\exp(-a_{oj}z_{okj}^2\tau) \\ + \frac{\alpha_{oj}a_{oj}}{\lambda_{oj}}\cos\left(\frac{1}{2}l_{oj}z_{okj}\right)\int_0^{\tau} t_o(\theta)\exp(-a_{oj}z_{okj}(\tau-\theta))\,d\theta\Bigg]. \end{aligned} \tag{12}$$

Here z_{okj} are the positive roots of the equation

$$\operatorname{tg}\left(\frac{1}{2}l_{oj}z\right) = \frac{\alpha_{oj}}{\lambda_{oj}z}. \tag{13}$$

The functions $t_o(\tau)$ and $t_i(\tau)$ can be approximated with a sufficient degree of accuracy by first-order splines. As is well known [4], in this case, the splines representing $t_o(\tau)$ and $t_i(\tau)$ can be obtained in terms of the B splines as follows:

$$t^n_o(\tau) = h \sum_{i=1}^{n} t_{oi} B_1(\tau - (i - 1) h), \tag{14}$$

$$t^n_i(\tau) = h \sum_{i=1}^{n} t_{ii} B_1(\tau - (i - 1) h), \tag{15}$$

where $B_1(\tau)$ are first-order splines relative to the nodes 0, h, 2h; h is the discretization interval and $0 \leqslant \tau \leqslant nh$.

It is not difficult to see that $t_o(ih) = t_{oi}$, $i = 0, 1, \ldots, n$; $t_i(ih) = t_{ii}$, $i = 0, 1, \ldots, n$.

When splines of order $m = 1$ $(m > 2)$ are used, Eqs. (14) and (15) assume the form

$$t^n_{i(\tau)} = h \sum_{i=-m}^{n} c_i B_{m-1,i}(\tau), \quad 0 \leqslant \tau \leqslant nh, \tag{16}$$

where $B_{m-1,i}(\tau)$ is the B spline of degree $(m - 1)$ relative to the nodes [4] ih, (i + 1)h, ..., (i + m + 1)h.

In order to determine the coefficients c_i, it is necessary to solve a system of linear equations which is inconvenient when n increases.

Considering the fact that the quantity $t_i(\tau)$ is measured with some error, it is useful to smooth the splines. However, their construction using the technique described in [4] is not a simple problem. In order to perform the smoothing, it is convenient to use the method of local approximation of functions by splines [5]. In this case, the coefficients c_i calculated quite simply, for example, we can set $c_i \doteq t_{ii}$; in this case, the accuracy of the approximation of the function $t_i(\tau)$ using Eq. (16) will be of order $\frac{h^2}{6} t''_i(\tau) + O(h^4)$ [5].

A more accurate approximation can be obtained by, for example, setting $c_i = -\frac{1}{6} t_{i(i-1)} + \frac{4}{3} t_{ii} - \frac{1}{6} t_{i(i+1)}$ [5]. The spline constructed in this manner is especially convenient when information on $t_i(\tau)$ and $t_o(\tau)$ contains noise.

It is clear that $t_{iwj}(\tau) = T_{ij}(0, \tau)$, $t_{oj}(\tau) = T_{oj}\left(\frac{l_{oj}}{2}, \tau\right)$ and $t_{iwj}(\tau)$ can be represented in the form of three terms: the first will arise from the effect of initial conditions, the second arises from the effect of the outdoor temperature, and the third arises from the indoor temperature. In a similar manner $t_{owj}(\tau)$ is represented as a sum of two terms: the first arises from the initial conditions and the second from the indoor temperature.

We shall denote these terms of $t_o(\tau) = t_i(\tau) = B_1(\tau)$ as follows:

$$\begin{gathered} t_{icj}(\tau) = \Delta t^{iy}_{iwj}(\tau) + \Delta t^{i}_{iwj}(\tau) + \Delta t^{o}_{iwj}(\tau), \\ t_{owj}(\tau) = \Delta t^{iy}_{owj}(\tau) + \Delta t^{o}_{owj}(\tau). \end{gathered} \tag{17}$$

Using the principle of superposition, it is not difficult to find that $t_{iwj}(\tau)$ for $t_o(\tau)$ and $t_i(\tau)$, represented by Eqs. (14) and (15):

$$t^n_{iwj}(\tau) = \Delta t^{iy}_{iwj}(\tau) + h \sum_{i=1}^{n} t_{ii} \Delta t^{i}_{iwj}(\tau - (i - 1) h) + h \sum_{i=1}^{n} t_{oj} \Delta t^{o}_{iwj}(\tau - (i - 1) h). \tag{18}$$

Similarly,

$$t^n_{owj}(\tau) = \Delta t^{iy}_{owj}(\tau) + h \sum_{i=1}^{n} t_{oi} \Delta t^{o}_{owj}(\tau - (i - 1) h). \tag{19}$$

Analysis of solutions (18) and (19) leads to the conclusion that in order to obtain them, the system of equations (2)-(9) must be solved only once in order to obtain the functions $\Delta t^{iy}_{iwj}(\tau)$, $\Delta t^{o}_{iwj}(\tau)$, $\Delta t^{i}_{iwj}(\tau)$, $\Delta t^{iy}_{owj}(\tau)$, $\Delta t^{o}_{owj}(\tau)$. Increasing the amount of starting data (i.e., the number n) does not make it necessary to perform the calculations using Eqs. (10) and (12) again.

For convenience in performing the calculations, we shall obtain the recurrence equations for $t^n_{iwj}(\tau)$, $t^n_{owj}(\tau)$:

TABLE 1. Thermophysical Parameters

Quantity	Value	Quantity	Value
F_0, m^2	4,0	α_{o2}, $W/m^2\cdot deg$	9,86
h_0, $W/m^2\cdot deg$	2,32	α_{io1}, $W/m^2\cdot deg$	9,86
a_{i1}, m^2/sec	$1,67\cdot 10^{-6}$	α_{io2}, $W/m^2\cdot deg$	9,86
a_{i2}, m^2/sec	$2,5\cdot 10^{-6}$	α_{io1}, $W/m^2\cdot deg$	29,0
a_{o1}, m^2/sec	$1,67\cdot 10^{-6}$	α_{io2}, $W/m^2\cdot deg$	29,0
a_{o2}, m^2/sec	$0,28\cdot 10^{-6}$	λ_{i1}, $W/m\cdot deg$	2,32
F_{i1}, m^2	20	λ_{i2}, $W/m\cdot deg$	1,16
F_{i2}, m^2	10	λ_{o1}, $W/m\cdot deg$	2,32
F_{o1}, m^2	40	λ_{o2}, $W/m\cdot deg$	3,48
F_{02}, m^2	10	l_{i1}, m	0,4
V, m^3	100	l_{i2}, m	0,5
c, $kJ/m^3\cdot deg$	1,045	l_{01}, m	0,15
α_{o1}, $W/m^2\cdot deg$	9,86	l_{02}, m	0,15

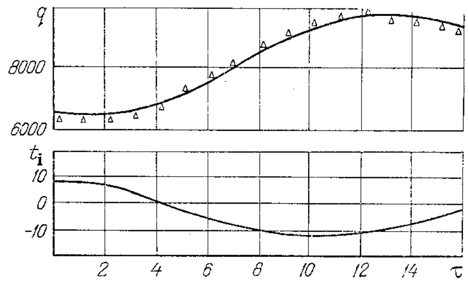

Fig. 1. Comparison of computational results: the curves follow the proposed method; the points follow the procedure in [2]. q, kJ; t_i, °C.

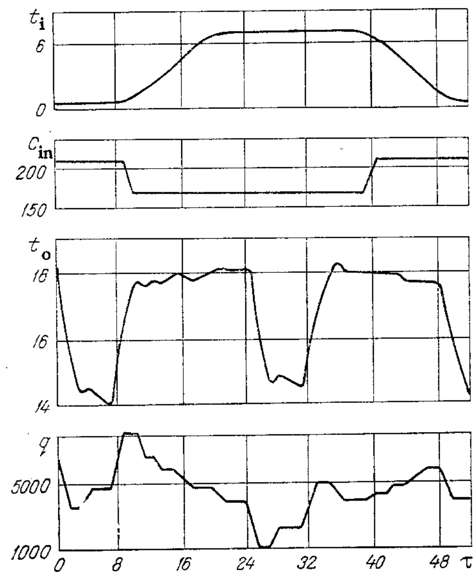

Fig. 2. The indoor temperature conditions with programmed heat release. C_{in}, kJ/h·deg; q, kJ/h.

$$t^{n+1}_{iwj}(\tau) = t^{n}_{iwj}(\tau) + ht_{i\,(n+1)}\Delta t^{i}_{iwj}(\tau - nh) + ht_{o\,(n+1)}\Delta t^{o}_{iwj}(\tau - nh); \tag{20}$$

$$t^{1}_{iwj}(\tau) = \Delta t^{iy}_{iwj}(\tau) + ht_{i1}\,\Delta t_{icj}(\tau) + ht_{o1}\Delta t^{o}_{iwj}(\tau); \tag{21}$$

$$t^{n+1}_{owj}(\tau) = t^{n}_{owj}(\tau) + ht_{w(n+1)}\Delta t^{w}_{owj}(\tau - nh); \tag{22}$$

$$t_{owj}(\tau) = \Delta t^{iy}_{owj}(\tau) + ht_{o1}\Delta t^{o}_{owj}(\tau). \tag{23}$$

Using the difference relation

$$\frac{dt_o(nh)}{d\tau} \approx \frac{t_o((n+1)h) - t_o(nh)}{h},$$

we obtain from Eq. (1)

$$t_{o(n+1)} = t_{on}\left(1 - \frac{h}{VC}\left(F_o k_o + C_{in}(nh) + \sum_j F_{ij}\alpha_{ioj} + \sum_j F_{oj}\alpha_{oj}\right)\right) + \frac{h}{VC_p} q(nh) + \frac{h}{VC_p}\left((F_o k_w + C_{in}(nh))\, t_{in} + \sum_j F_{ij}\alpha_{ioj} t^{n}_{iwj}(nh) + \sum_j F_{oj}\alpha_{oj} t^{n}_{owj}(nh)\right). \tag{24}$$

Eq. (24) permits calculating $t_{o_1}, t_{o_2}, \ldots, t_{on};\ t_{i_1}, t_{i_2}, \ldots, t_{in}$ using the values of the indoor and outdoor temperatures $C_{in}(ih)$, $q(ih)$, $i = 1, 2, \ldots, n$ and the known values of $t_{o(n+1)}$.

Thus, expressions (20)-(24) define an algorithm with whose help it is possible to calculate $t_o(\tau)$ for any n for known $q(\tau)$, $t_i(\tau)$, $C_{in}(\tau)$. Equations (20)-(24) can also be used to calculate the required amount of heat $q(\tau)$ for a programmed change in the indoor temperature.

Using the equations indicated above, we wrote a FORTRAN-IV program for the ES-1022 computer.

In order to check the adequacy of the working equations obtained, we examined the temperature regime in an enclosure consisting of two equivalent exterior and interior walls. The values of the thermophysical parameters used in the calculations are presented in Table 1.

The oscillations in the outdoor air temperature were taken as $t_i(\tau) = (-2) + 10\cos(0.261\,\tau)$, and the infiltration factor was taken as a constant $C_{in} = 0.048$ kJ/deg·sec; the indoor air temperature was $t_o = 18°C$. A comparison with the well-known computational procedure in [3] showed (Fig. 1) that the computational error does not exceed 5%.

In order to estimate the efficiency of the method proposed, we calculated the indoor air temperature as a function of the varying quantities C_{in}, t_i, q with the same thermophysical parameters (Fig. 2). The numerical experiments showed that with an operational definition of the parameters with an interval of 0.2 h, in the case of the solution obtained by operational methods [2], the expenditures of machine time constitute more than 3 h per day. The use of the method examined here decreased the machine time used to 10-15 min.

Thus, with the help of this method, it is possible to calculate heat transfer in buildings with arbitrary variation in the outside air and other perturbations and controlling parameters.

NOTATION

$t_o(\tau)$, interior air temperature averaged over the volume of the enclosure; $t_i(\tau)$, outside air temperature; $t_{iwj}(\tau)$, temperature of the internal surface of the j-th external wall; $t_{owj}(\tau)$, temperature of the surface of the j-th internal wall; $C_{in}(\tau)$, infiltration factor ($C_{in} = VnC_p$); $q(\tau)$, amount of heat, introduced into the enclosure; V, volume of the enclosure; C_p, heat capacity of air; F_w, area of the windows; k_w, coefficient of heat transfer through the windows; F_{oj}, F_{ij}, areas of the j-th inner and outer walls, respectively; α_{oj}, α_{ioj}, α_{iwj}, coefficients of heat transfer of the j-th inner and outer walls; l_{oj}, l_{ij}, thicknesses of the j-th inner and outer walls; λ_{oj}, λ_{ij}, coefficients of thermal conductivity of the j-th inner and outer walls; a_{oj}, a_{ij}, coefficients of thermal diffusivity of the j-th inner and outer walls; n, multiplicity of the air exchange; $T_{oj}(x)$, $T_{ij}(x)$, initial temperature distributions over the thickness in the j-th inner and outer walls.

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TEMPERATURE DEPENDENCE OF THE VELOCITY OF ULTRASOUND AND ELECTRICAL CONDUCTIVITY OF AN EPOXY COMPOSITE WITH A CARBON FILLER

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Experimental data are given on the behavior of the velocity of ultrasound and electrical conductivity as a function of the temperature in an epoxy composite material filled with carbon fiber.

Objective prerequisites have now been established for adaptation to the general industrial application of composite materials [1]. This fact necessitates comprehensive investigation of the properties of new materials to test their compliance with the set of requirements imposed on structural materials in various branches of engineering, the variations of the properties in service, aging, etc. The complexity of these systems makes it rather difficult to employ conventional research techniques such as, for example, chemical and spectroscopic procedures. In recent years, therefore, acoustical and electrophysical methods have begun to enjoy widespread application [2, 3], and multiparameter methods are being developed for the comprehensive investigation of composites with simultaneous measurement of their various properties.

We have studied the temperature dependence of the electrical conductivity and velocity of propagation of ultrasound in an epoxy composite material without filler and with a filler of powdered quartz and metal-infused carbon fiber (the lengths of the fiber segments were up to 1 mm). In the conventional classification scheme the investigated composites are statistical mixtures.

The electrical conductivity was measured by means of stainless-steel electrodes. The structure of the measurement cell ensured parallelism between the working surfaces of the electrodes and an invariant spacing between them during the experiment. The working surfaces of the electrodes were polished to a high degree of purity, minimizing adhesion of the material to the electrode.

The velocity of ultrasound in the hardened epoxy composite was measured by the buffer-rod method [4] with the application of continuous ultrasonic waves.

The measurement cells for determining the conductivity and velocity of ultrasound were placed in an air thermostat. A Chromel–Copel thermocouple and a PP-63 potentiometer were used to monitor the temperature of the material during the experiment. The resistance of the material was recorded with a Straton Teralin III instrument with a measurement range of 10^3-10^{16} Ω.

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